第十一章 基于形式化方法的服务质量保障

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软件形式化方法

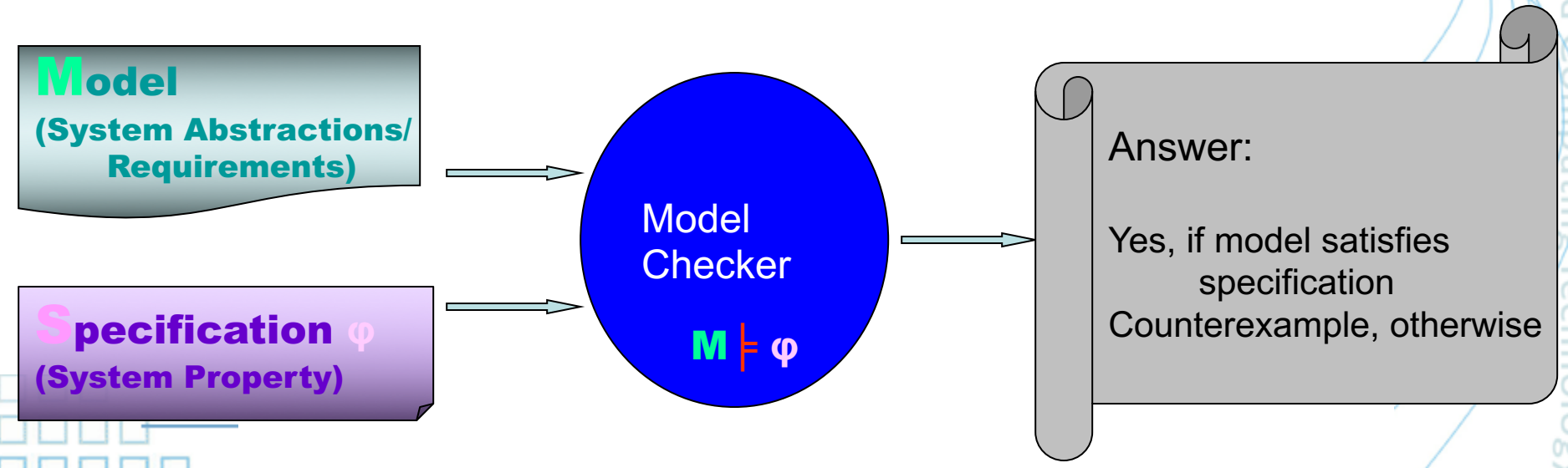
- 采用数学上的形式化方法对软件进行抽象和描述A
  - finite state machines，labelled transition systems，Petri nets, vector addition systems, timed automata, hybrid automata, process algebra等
- 目的是对某些属性进行验证和证明
  - 属性a：使用形式化方法进行描述
  - 推理或证明：A→a? If not, counterexample

形式化验证方法

- 一般来说，不同的软件形式化方法、不同的属性形式化方法对应不同的形式化验证方法
- 两种常见形式化验证方法
  - 模型检测(Model Checker)
    - 软件描述为状态和状态的变化，通过对所有状态的枚举或搜索来判断某些属性是否满足
  - 演绎推理(Deductive Verification)
    - 将软件实现抽象为一系列逻辑语句，将属性描述为定理逻辑语句，最后借用定理证明器(theorem provers)或SMT求解器等进行演绎推理，从而判断定理是否为真

模型检测

- 模型(Model)
  - 软件系统的抽象模型，一般是有限状态机
- 属性(Property)
  - 形式化的规约(Specification)



属性

- **安全属性Safety Properties**
  - **Invariants, deadlocks, reachability, etc.**
    - Invariants: “x is always less than 10”
    - Deadlock freedom: “the system never reaches a state where no moves are possible”
  - **“something bad never happens”**
- **活性属性Liveness Properties**
  - **Termination, response, etc.**
    - Termination: “the system eventually terminates”
    - Response properties: “if action X occurs then eventually action Y will occur”
  - **“something good will eventually happen”**

信号互斥例子(Mutual Exclusion)

Model
(System Requirements)

The Model (Willem Visser, http://ase.arc.nasa.gov/visser/ASE2002TutSoftwareMC-fonts.ppt)

- Two process mutual exclusion with shared semaphore
- Each process has three states
  - Non-critical (N)
  - Trying (T)
  - Critical (C)
- Semaphore can be available (S_0) or taken (S_1)
- Initially both processes are in the Non-critical state and the semaphore is available --- $N_1\, N_2\, S_0$

$$
\begin{array}{lll}
N_1 & \rightarrow & T_1 \\
T_1 \wedge S_0 & \rightarrow & C_1 \wedge S_1 \\
C_1 & \rightarrow & N_1 \wedge S_0
\end{array}
\quad \| \quad
\begin{array}{lll}
N_2 & \rightarrow & T_2 \\
T_2 \wedge S_0 & \rightarrow & C_2 \wedge S_1 \\
C_2 & \rightarrow & N_2 \wedge S_0
\end{array}
$$

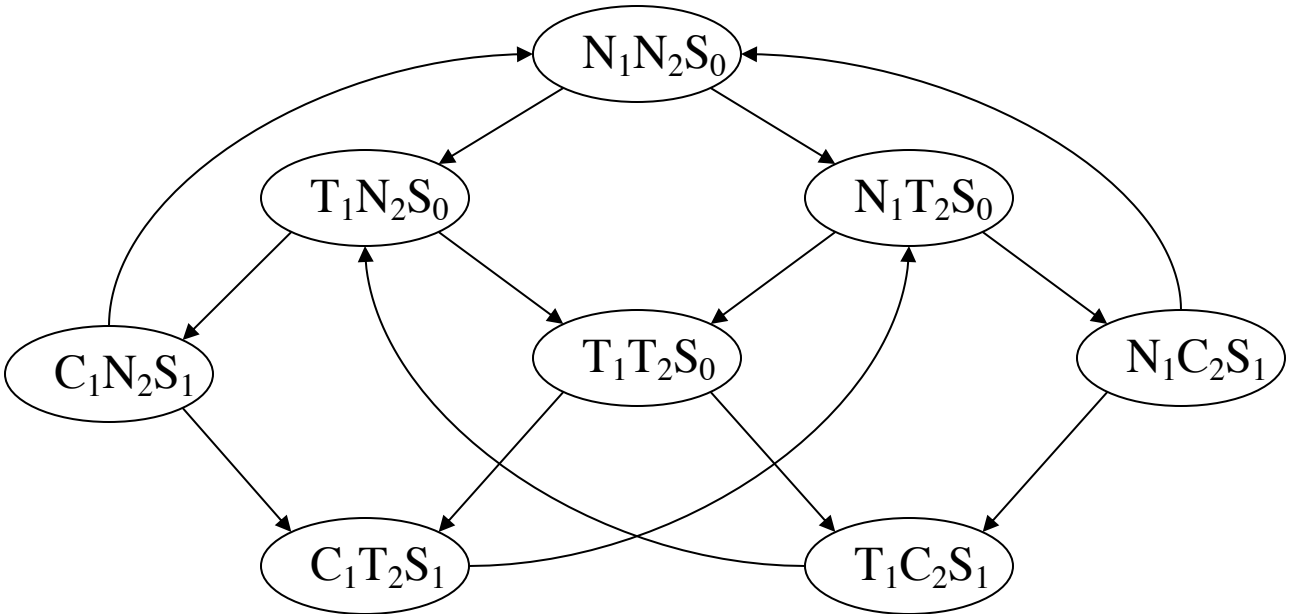
信号互斥例子(Mutual Exclusion)

Model
(System Requirements)

The Model (Willem Visser, http://ase.arc.nasa.gov/visser/ASE2002TutSoftwareMC-fonts.ppt)

- Initially both processes are in the Non-critical state and the semaphore is available --- $N_1\, N_2\, S_0$

$$\begin{array}{lll} N_1 & \rightarrow & T_1 \\ T_1 \wedge S_0 & \rightarrow & C_1 \wedge S_1 \\ C_1 & \rightarrow & N_1 \wedge S_0 \end{array} \quad \| \quad \begin{array}{lll} N_2 & \rightarrow & T_2 \\ T_2 \wedge S_0 & \rightarrow & C_2 \wedge S_1 \\ C_2 & \rightarrow & N_2 \wedge S_0 \end{array}$$



信号互斥例子(Mutual Exclusion)

Specification
(System Property)

Specification – Desirable Property

No matter where you are there is always a way to get to the initial state

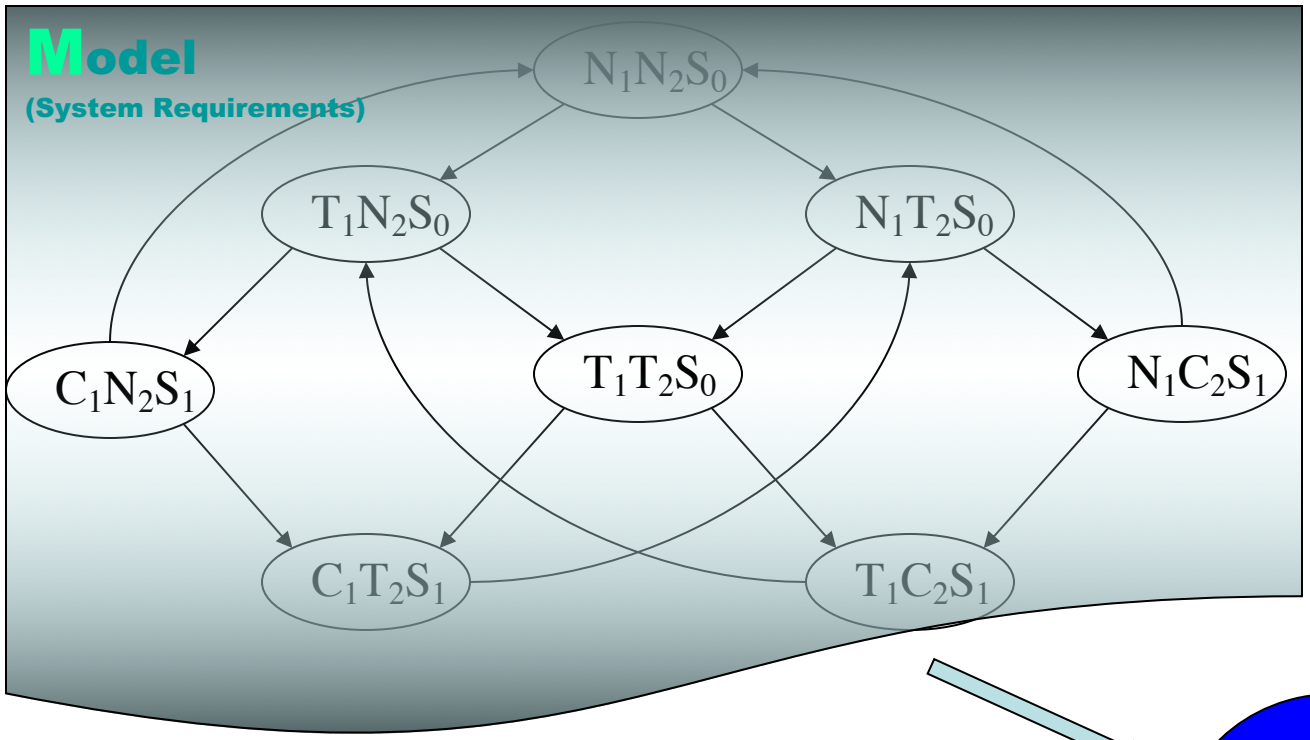
State Machine $\models$ AG EF $(N_1 \wedge N_2 \wedge S_0)$

CTL (Computation Tree Logic)

$M \models \varphi$

- Quantifiers over paths
  - **A** φ – **A**ll: φ has to hold on all paths starting from the current state.
  - **E** φ – **E**xists: there exists at least one path starting from the current state where φ holds.
- Path-specific quantifiers
  - **X** φ – Next: φ has to hold at the next state (this operator is sometimes noted **N** instead of **X**).
  - **G** φ – **G**lobally: φ has to hold on the entire subsequent path.
  - **F** φ – **F**inally: φ eventually has to hold (somewhere on the subsequent path).

信号互斥例子(Mutual Exclusion)



Model Checker

$M \models \varphi$

Answer: Yes

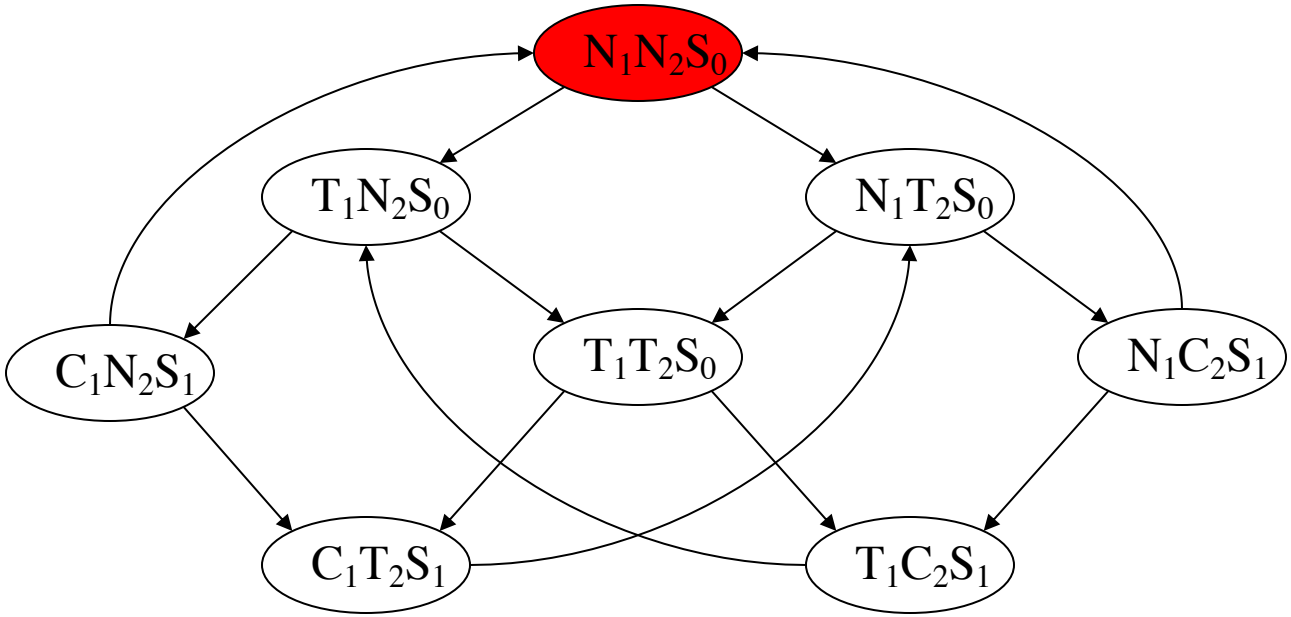
Specification
(System Property)

$$M \models AG\ EF\ (N_1 \wedge N_2 \wedge S_0)$$

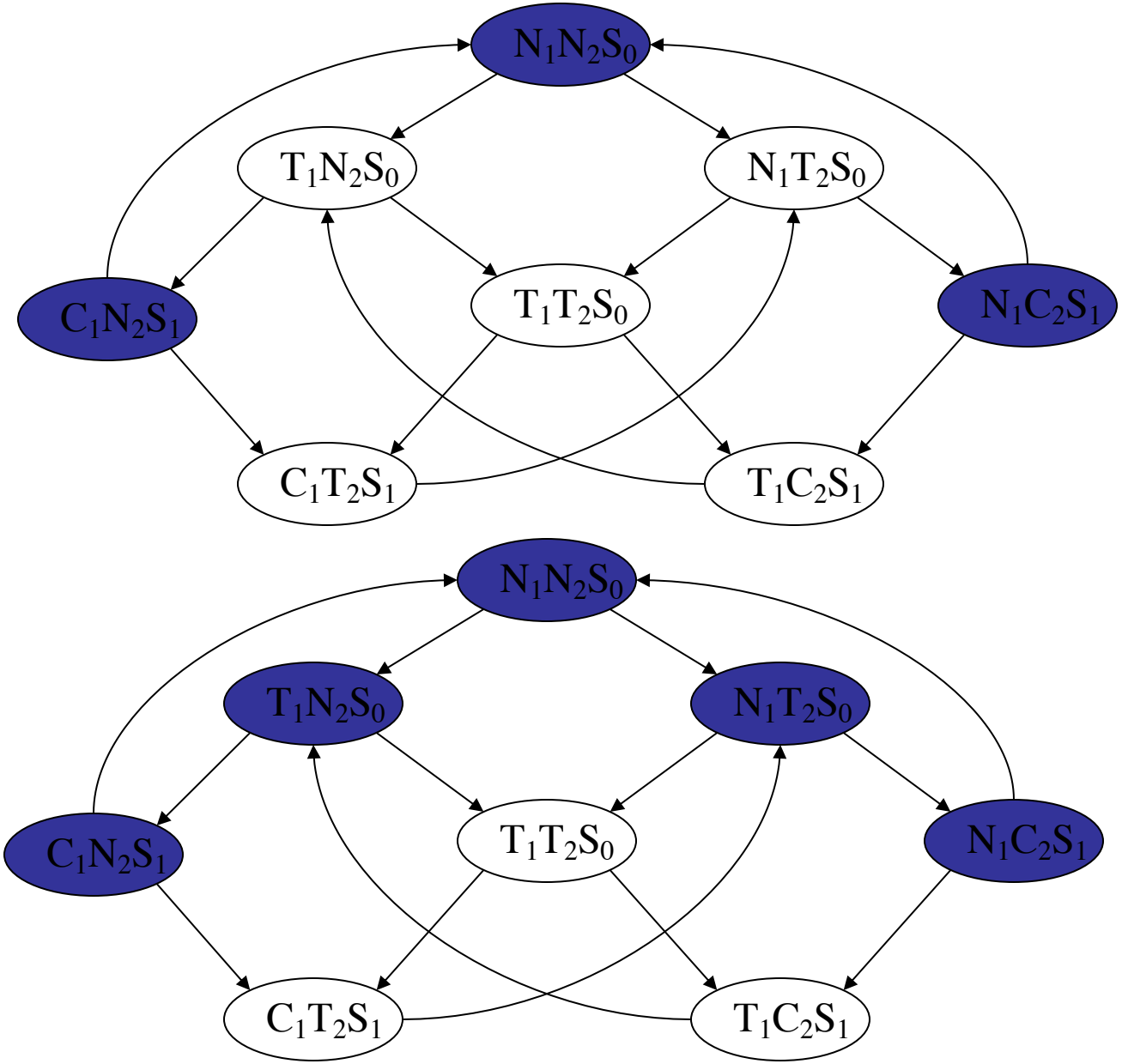
信号互斥例子(Mutual Exclusion)

Answer: Yes

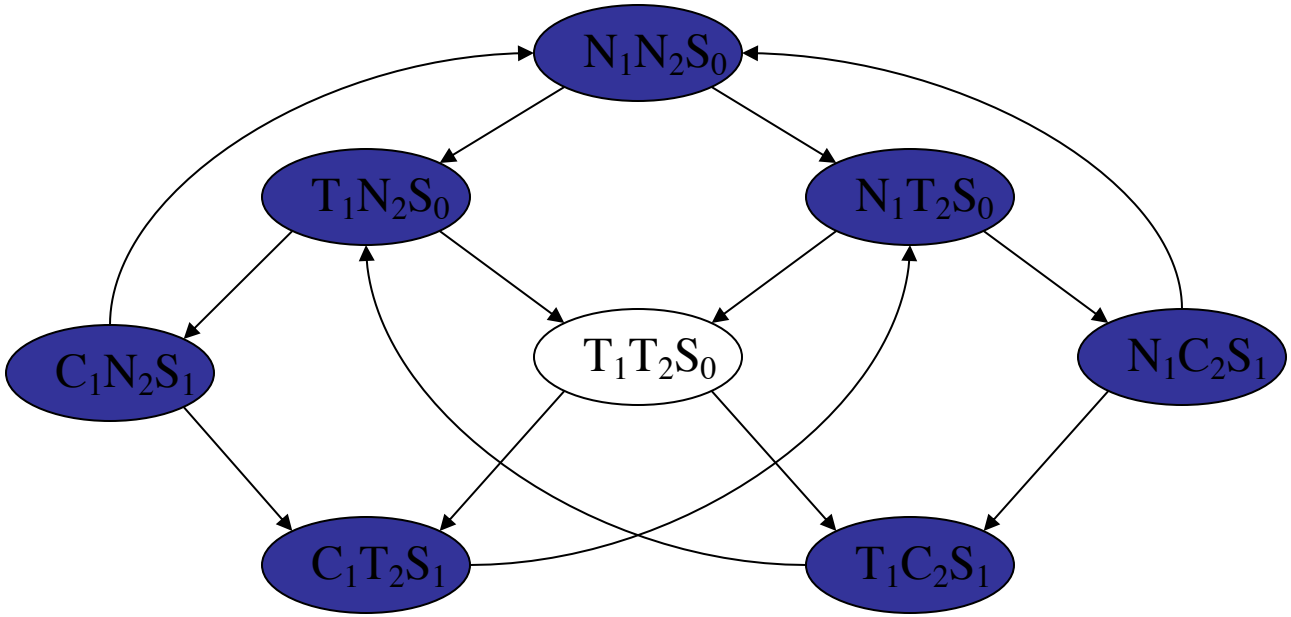
A Proof: *For All* possible behaviors

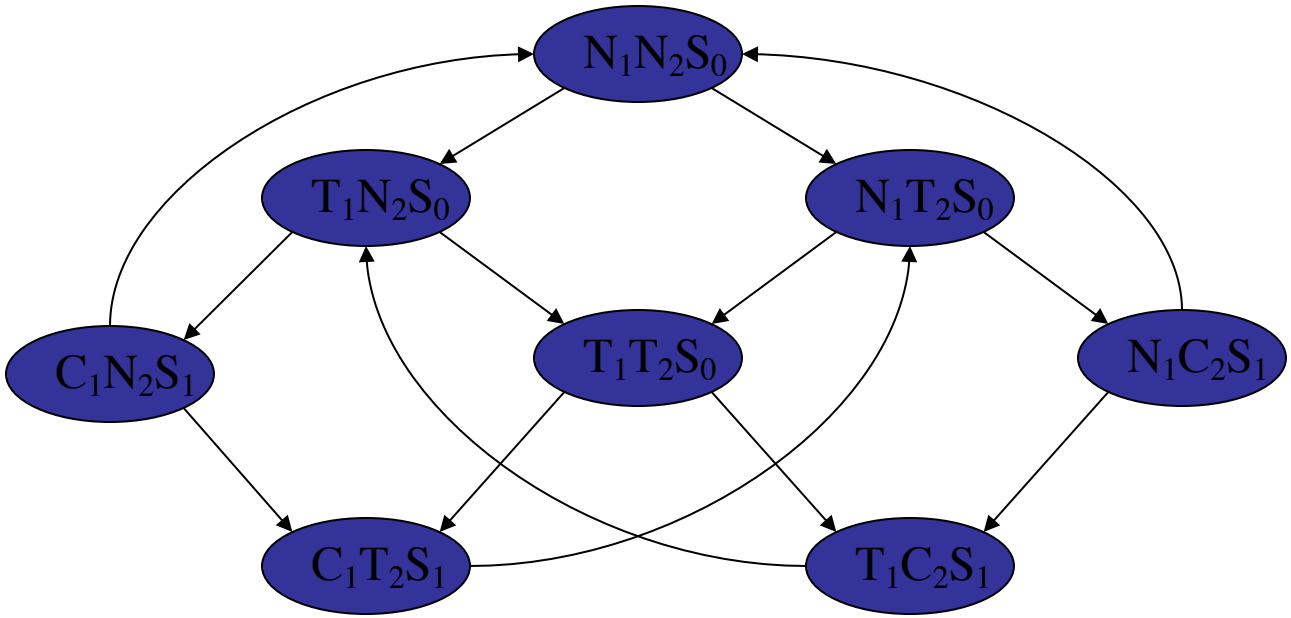


信号互斥例子(Mutual Exclusion)



信号互斥例子(Mutual Exclusion)





信号互斥例子(Mutual Exclusion)

Specification – Desirable Property

No matter where you are there is

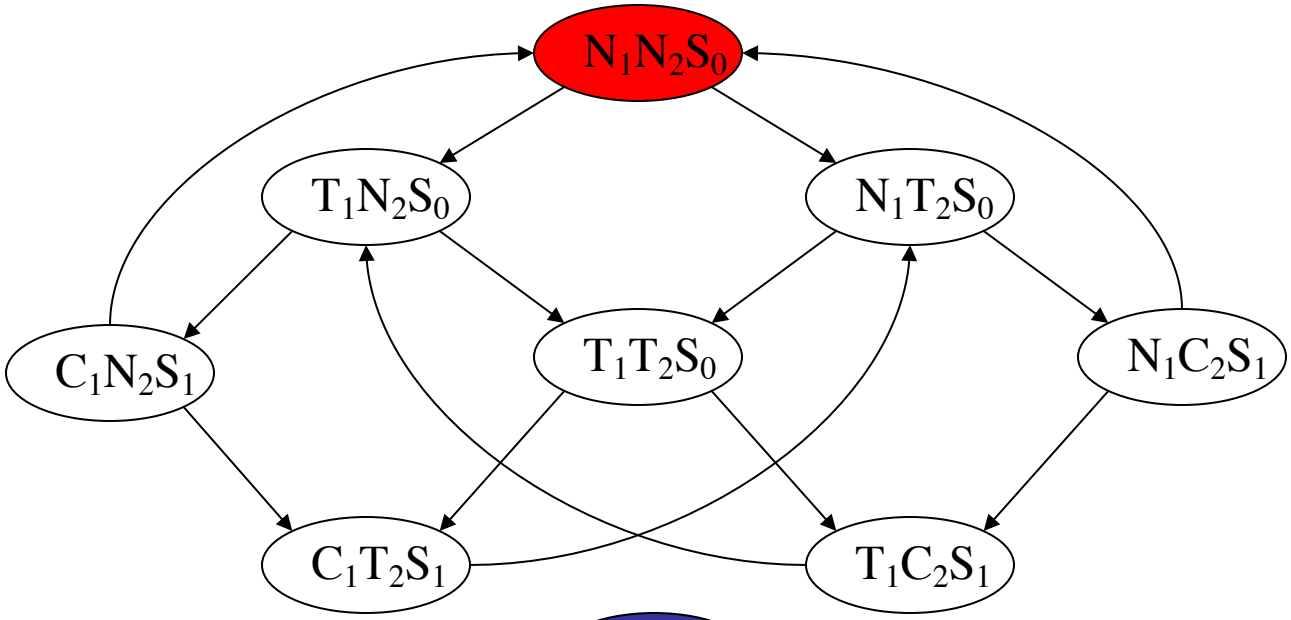
no *way to get to the initial state*

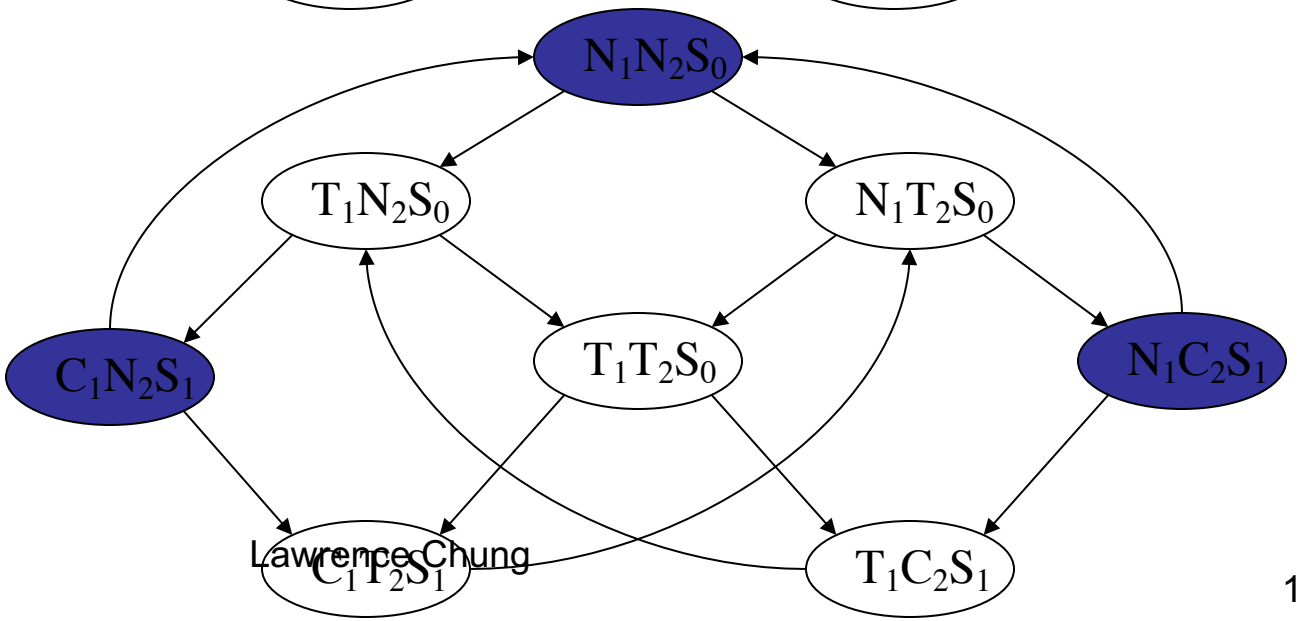
$$\neg \; M \models AG\, EF\, (N_1 \wedge N_2 \wedge S_0)$$

信号互斥例子(Mutual Exclusion)

Answer: No

Counterexample





模型形式化方法

- 系统模型：状态机及其扩展
- Kripke Structures

Let AP be a set of labels — i.e., a set of atomic propositions such as Boolean expressions over variables, constants, and predicate symbols.

A Kripke structure is a 4-tuple, $M = (S, I, R, L)$:

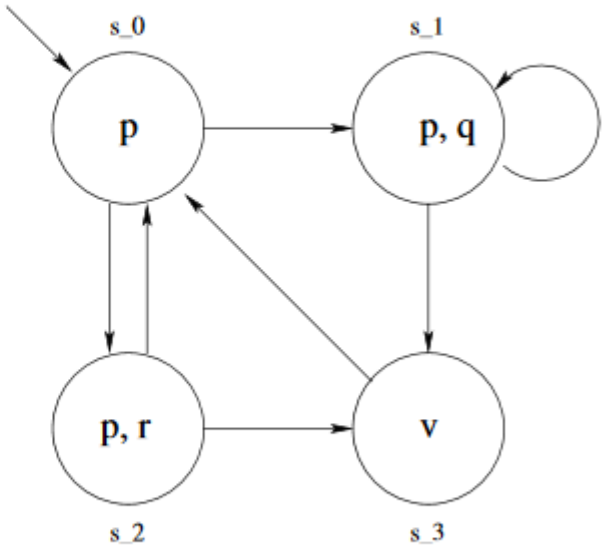
- a finite set of states, S,
- a set of initial states, $I \subseteq S$,
- a transition relation, $R \subseteq S \times S$ where $\forall s \in S, \exists s' \in S$ such that $(s, s') \in R$,
- a labeling function, L, from states to the power set of atomic propositions, $L : S \rightarrow 2^{AP}$.

Kripke Structures例子

$S = \{s_0, s_1, s_2, s_3\}$
$I = \{s_0\}$

$$R = \{\{s_0, s_1\}, \{s_0, s_2\}, \{s_1, s_1\}, \{s_1, s_3\}, \{s_2, s_0\}, \{s_2, s_3\}, \{s_3, s_0\}\}$$

$$L = \{\{s_0, \{p\}\}, \{s_1, \{p, q\}\}, \{s_2, \{p, r\}\}, \{s_3, \{v\}\}\}$$

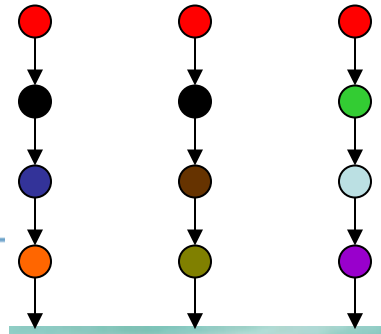


属性描述方法

- **时序逻辑(Temporal Logic)**
  - **描述事件的时序属性**
  - **“Always”，“Eventually”**

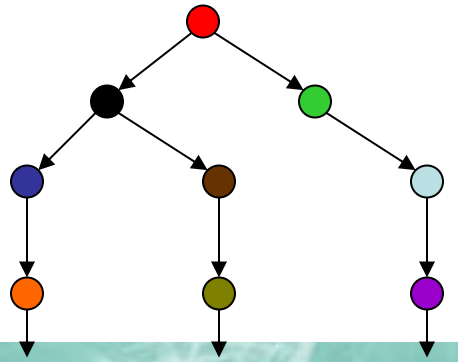
❖Linear Time

- **Every moment has a unique successor**
- **Infinite sequences (words)**
- **Linear Time Temporal Logic (LTL)**



❖Branching Time

- Every moment has several successors
- Infinite tree
- Computation Tree Logic (CTL)



模型检测方法和工具：TLA+

- TLA(Temporal Logic of Actions)
  - 一种描述LTL变体的规范(Specification)
- TLA+: A TLA IDE Model Checking Tool
  - 模型检测工具
  - 可定义状态机、可描述待验证的属性
  - 可自动推理验证属性是否为真；如为假，则自动给出反例
  - 图灵奖获得者Lamport设计
- 主页：https://lamport.azurewebsites.net/tla/tla.html

TLA+例子(1)

- **Die Hard Problem**
  - 有两个水壶：3 gallons，5 gallons
  - 一个水龙头可以出水
  - 水壶可以接满水
  - 水壶之间可以倾倒水

- 问题：采用什么方法使得大水壶里有且仅有4 gallons水？可行吗？

TLA+例子(2)

- 模型：状态机定义

- Actions
  - 水壶灌满水
  - 水壶倒空
  - 水壶间倒水

```
VARIABLES small, big

Init == /\ big   = 0 /\ small = 0

FillSmall == /\ small' = 3 /\ big'   = big

FillBig == /\ big'   = 5 /\ small' = small

EmptySmall == /\ small' = 0 /\ big'   = big

EmptyBig == /\ big'   = 0  /\ small' = small

SmallToBig == IF big + small =< 5
              THEN /\ big'   = big + small
                   /\ small' = 0
              ELSE /\ big'   = 5
                   /\ small' = small - (5 - big)

BigToSmall == IF big + small =< 3
              THEN /\ big'   = 0
                   /\ small' = big + small
              ELSE /\ big'   = small - (3 - big)
                   /\ small' = 3

Next == \/ FillSmall
        \/ FillBig
        \/ EmptySmall
        \/ EmptyBig
        \/ SmallToBig
        \/ BigToSmall
```

TLA+例子(3)

- 定义属性：Big4 == big /= 4
  - big变量always不等于4
- TLA+自动验证Big4属性
  - 如为真，说明不存在一种方法使得大水壶中水量为4
  - 否则，给出一种方案使得big=4

TLA+例子(4)

TLA Module

```
4  VARIABLES small, big
5
6  TypeOK == /\ small \in 0..3
7            /\ big   \in 0..5
8
9  Big4 == big /= 4
10
11 Init == /\ big   = 0 /\ small = 0
12
13 FillSmall == /\ small' = 3 /\ big'   = big
14
15 FillBig == /\ big'   = 5 /\ small' = small
16
17 EmptySmall == /\ small' = 0 /\ big'   = big
18
19 EmptyBig == /\ big'   = 0  /\ small' = small
20
21 SmallToBig == IF big + small =< 5
22               THEN /\ big'   = big + small
23                    /\ small' = 0
24               ELSE /\ big'   = 5
25                    /\ small' = small - (5 - big)
26
27 BigToSmall == IF big + small =< 3
28               THEN /\ big'   = 0
29                    /\ small' = big + small
30               ELSE /\ big'   = small - (3 - big)
31                    /\ small' = 3
32
33 Next == \/ FillSmall
34        \/ FillBig
```

Invariant Big4 is violated.

Error-Trace Exploration

Error-Trace

Name	Value
small	0
<Action line 27, col 15 to line 31, col 33 o	State (num = 3)
big	2
small	3
<Action line 17, col 15 to line 17, col 43 o	State (num = 4)
big	2
small	0
<Action line 27, col 15 to line 31, col 33 o	State (num = 5)
big	0
small	2
<Action line 15, col 12 to line 15, col 42 o	State (num = 6)
big	5
small	2
<Action line 27, col 15 to line 31, col 33 o	State (num = 7)
big	4
small	3

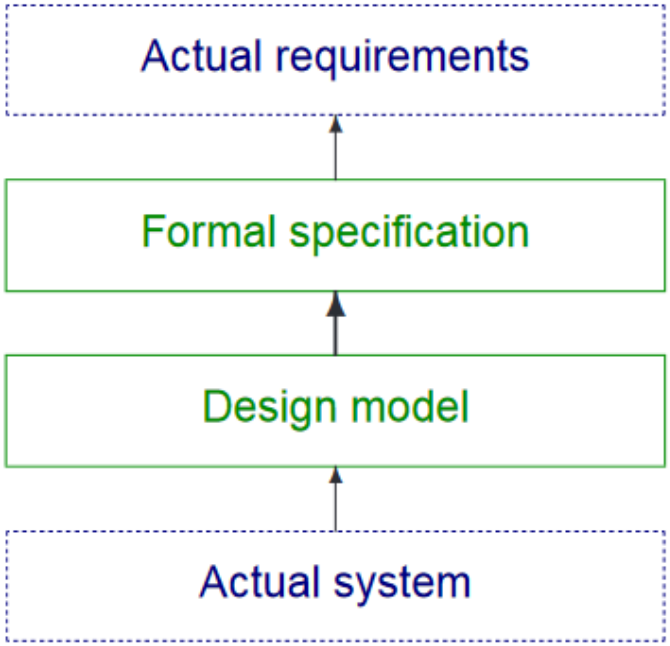
其余模型检测方法和工具

- 自定义状态机
- 直接使用LTL和CTL的语法
- 自己设计状态空间搜索算法

- 其它工具
  - SPIN：LTL
  - LTSmin： LTL, CTL*(CTL extension)
  - 具体查看 https://en.wikipedia.org/wiki/List_of_model_checking_tools

演绎推理

- 定理证明和推理
  - 设计模型：从真实代码中抽象出来
  - 形式化需求规约：真实需求的形式化表示
  - 证明：设计模型是否符合形式化需求规约



如何形式化？

- 形式化设计模型和需求规约
- 逻辑方法
  - 命题逻辑(布尔代数, Proposition Logic)) PL
  - 一阶逻辑(First-order Logic) FOL
  - 高阶逻辑(High-order Logic) HOL
- 形式化语言的表达能力越强，后续的证明过程自动化的难度越大

命题逻辑

- 原子命题p, q
- 命题公式：$p \vee \neg p$, $\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$

- 证明过程是可满足性问题(SAT)
  - 第一个被证明的NP完全问题
  - 除非P=NP，不存在线性的自动推理方法
- 实际算法：足够好
  - Davis-Putnam-Loveland-Logemann (DPLL)
  - Stalmarck's method

一阶逻辑

- 变量和函数: x, y, f(x)
- 算术公式：x+1 < 2y, R(f(x),y)
- 量词： ∀x∃y(f(x))
- 证明过程是FOL验证问题
  - 不可判定问题，半可判定
  - 为真(即定理），存在算法最终可验证；为假，可能一直计算无法判断
  - 不存在有效的验证算法
- 去掉量词的一阶逻辑： 存在有效验证算法SMT

高阶逻辑

- 在一阶逻辑基础上，量词可作用于谓词和函数：
  - $\forall P.\ P(0) \wedge (\forall n.\ P(n) \Rightarrow P(n+1)) \Rightarrow \forall n.\ P(n)$
    - 自然数：同构于二阶逻辑公理系统
- 证明过程是HOL验证问题
  - 甚至不是半可判定的
  - 不存在有效的验证算法

类型理论(Type Theory)

- 类型理论
  - 一种形式化系统：包括类型以及类型上的操作
  - 典型代表：Calculus of (inductive) constructions (CoC, CIC)，即typed lambda calculus
- HOL的量词可作用于谓词和函数
  - 可看作是类型以及高阶的类型
  - CoC可用于描述HOL
- 一般认为，CoC和HOL的表达能力差不多，在实际使用上不严格区分

软件交互式定理证明

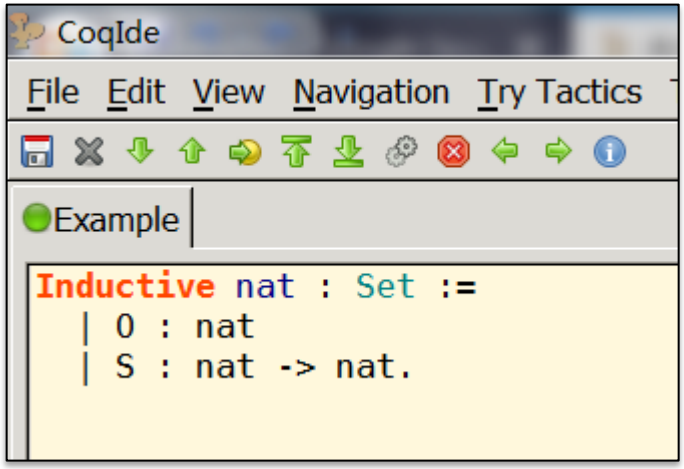
- HOL甚至FOL不存在有效的自动推理工具
  - 命题逻辑：除非P=NP，不存在线性的自动推理方法
  - FOL：半可判定
  - HOL：甚至不是半可判定
- 交互式的定理证明(Interactive Theorem Prover)
  - 人工辅助的方式进行快速推理
  - 使之成为实用且有效的定理证明工具

交互式定理证明工具

- **Coq工具：** **https://coq.inria.fr/**
  - 基于CoC，可用于描述HOL和Typed函数式编程
- **HOL工具：** **https://hol-theorem-prover.org/**
  - 描述HOL
- **Isabelle工具：**https://isabelle.in.tum.de/
  - 描述HOL
- 即使最易用的证明器，也不是一件容易的事情
  - 需要人工进行形式化
  - 需要人工交互和干预
- 原则：让能够自动化的，使它自动化

Coq工具例子(1)

- Peano算术公理系统：两条核心公理
  - 0 is a natural number.
  - For every natural number n, (S n) is a natural number.



- 0 is 0
- 1 is (S 0) = (1 + 0)
- 2 is (S (S 0)) = (1+ (1 + 0))
- 3 is (S (S (S 0)) = (1 + (1 + (1 + 0)))
- ...

Coq工具例子(2)

- 定义算术加法:递归定义函数

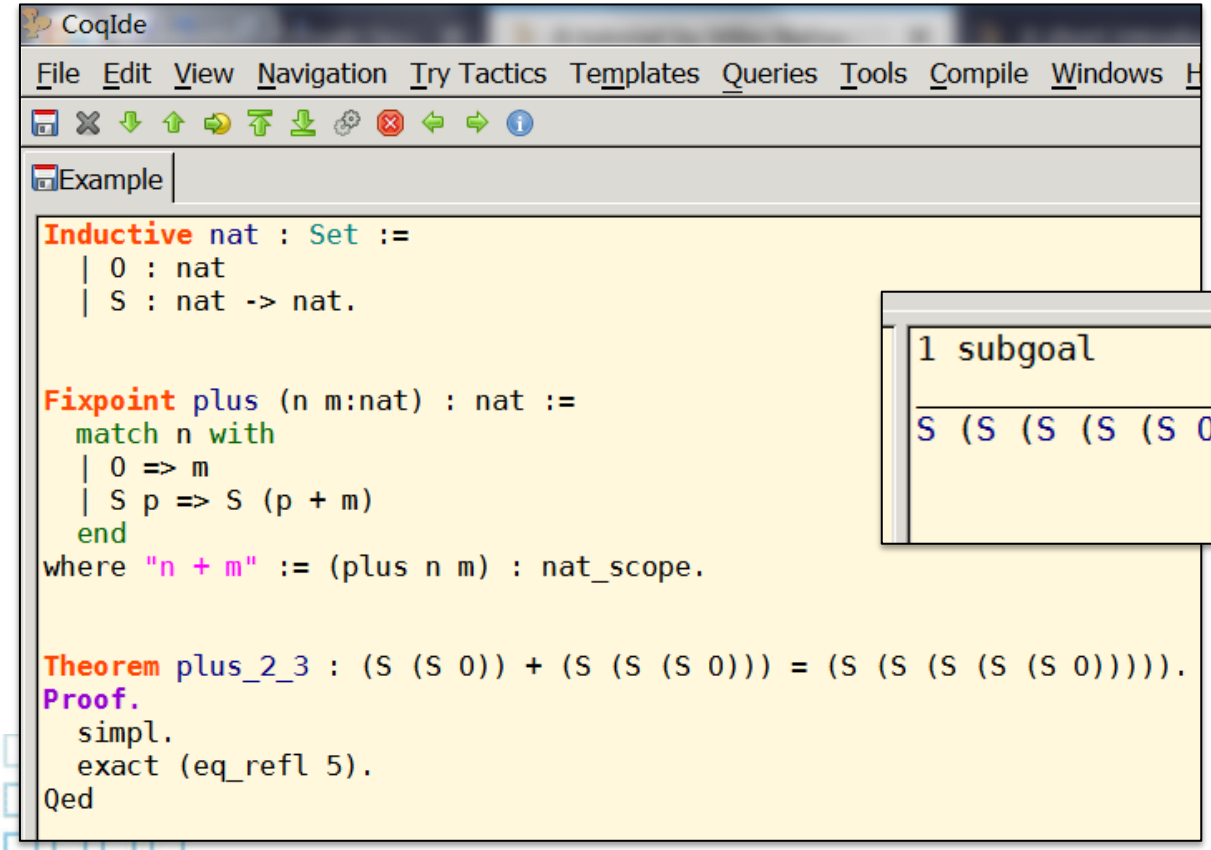
```
Fixpoint plus (n m:nat) : nat :=
  match n with
  | O => m
  | S p => S (p + m)
  end
where "n + m" := (plus n m) : nat_scope.
```

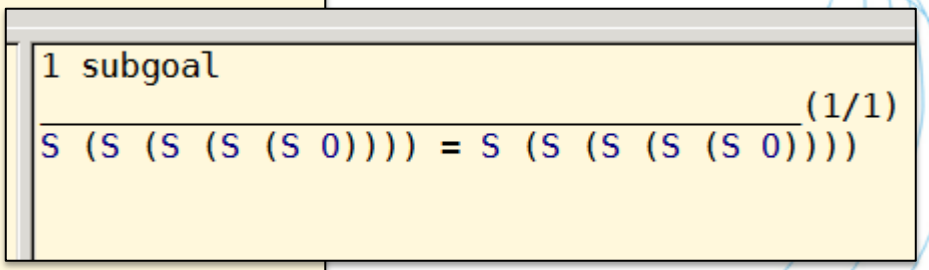
- 2+3=? 手动推理过程：

  - (plus (S (S O)) (S (S (S O))))
  - (S (plus (S O) (S (S (S O)))))
  - (S (S (plus O (S (S (S O))))))
  - (S (S (S (S (S O)))))

Coq工具例子(3)

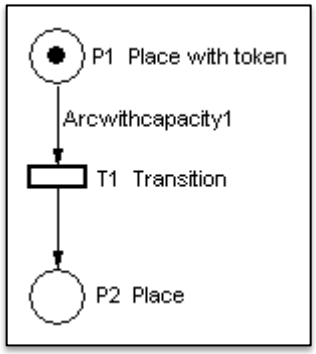
- 定理自动证明

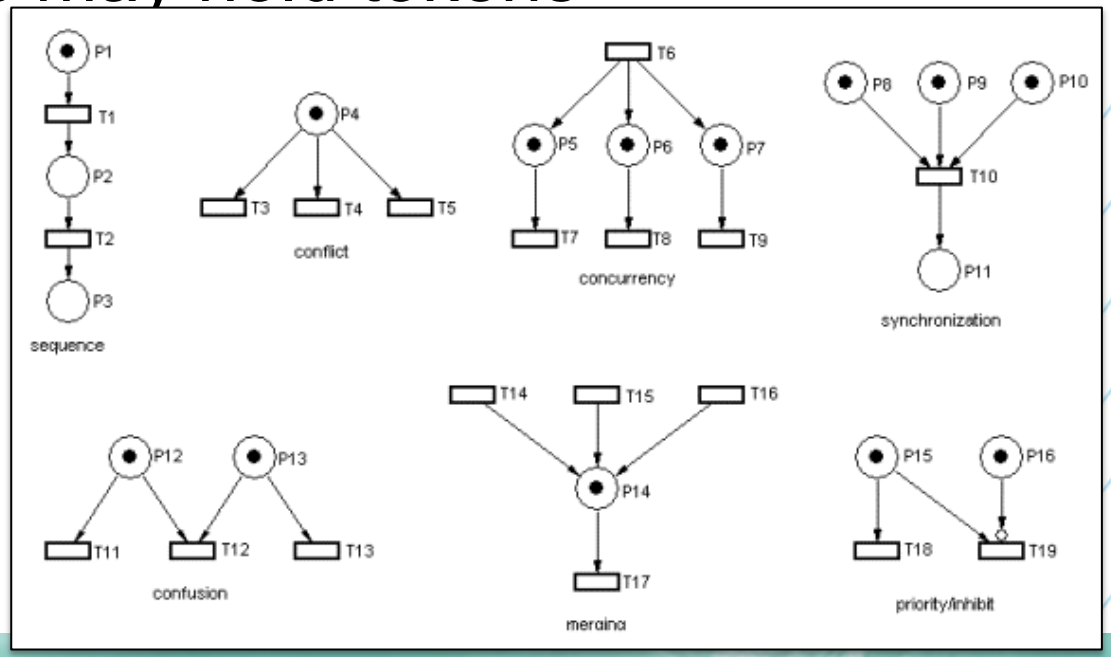




其他方法：Petri Nets

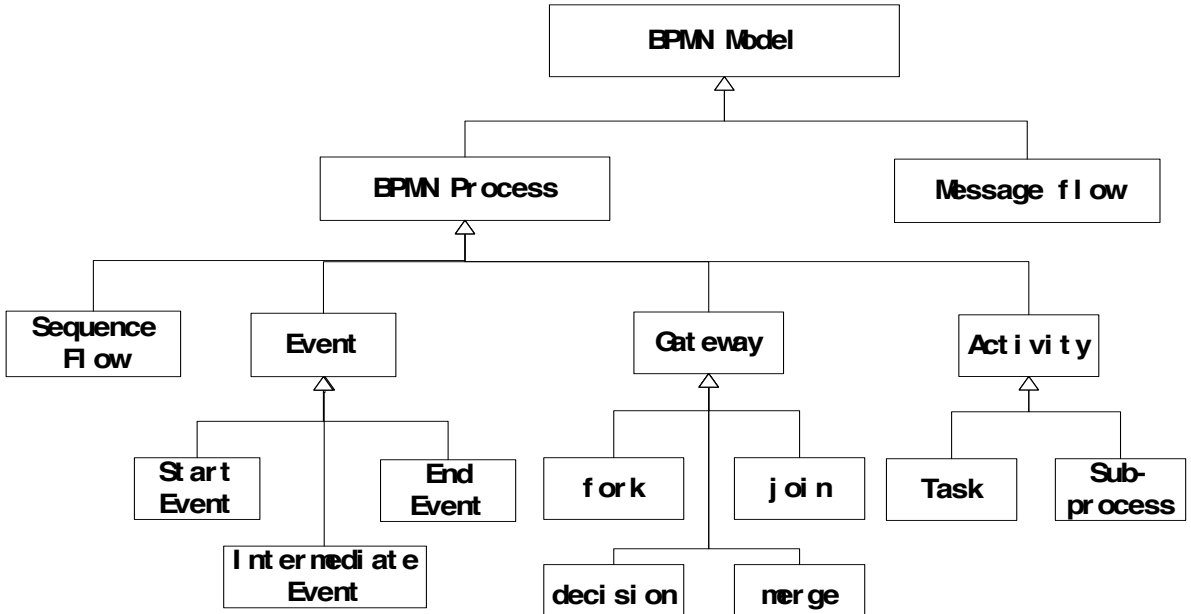
- **Petri Net:**
  - a collection of directed arcs connecting places(库所) and transitions(变迁)
  - Places may hold tokens



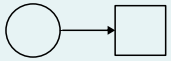
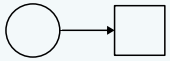
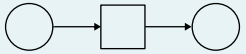
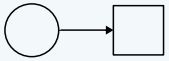
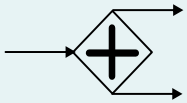
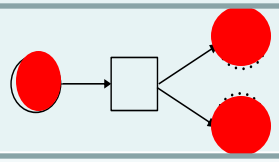
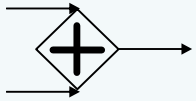
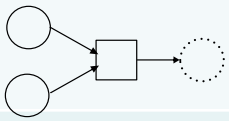
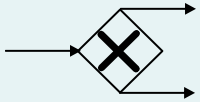
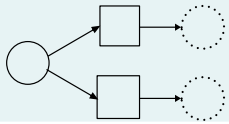
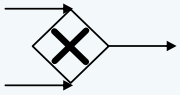
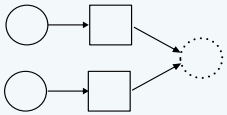


基于Petri网的组合服务验证

- 组合服务用业务流程模型BPMN表示
- 一个Core BPMN Model是一个二元组CBM＝（$\mathcal{Q}$,$\mathcal{FM}$），其中：
  - $\mathcal{Q}$是核心BPMN流程CBP的集合
  - $\mathcal{FM}$ 是流程之间的消息流message Flow的集合
- 一个Core BPMN Process是一个五元组 CBP=($\mathcal{O}$, $\mathcal{T}$, $\mathcal{E}$, $\mathcal{G}$, $\mathcal{F}$)

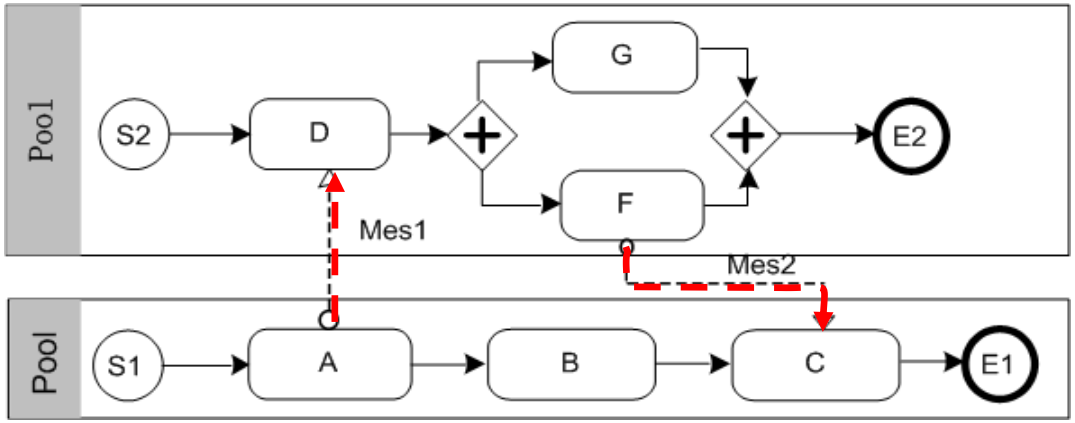


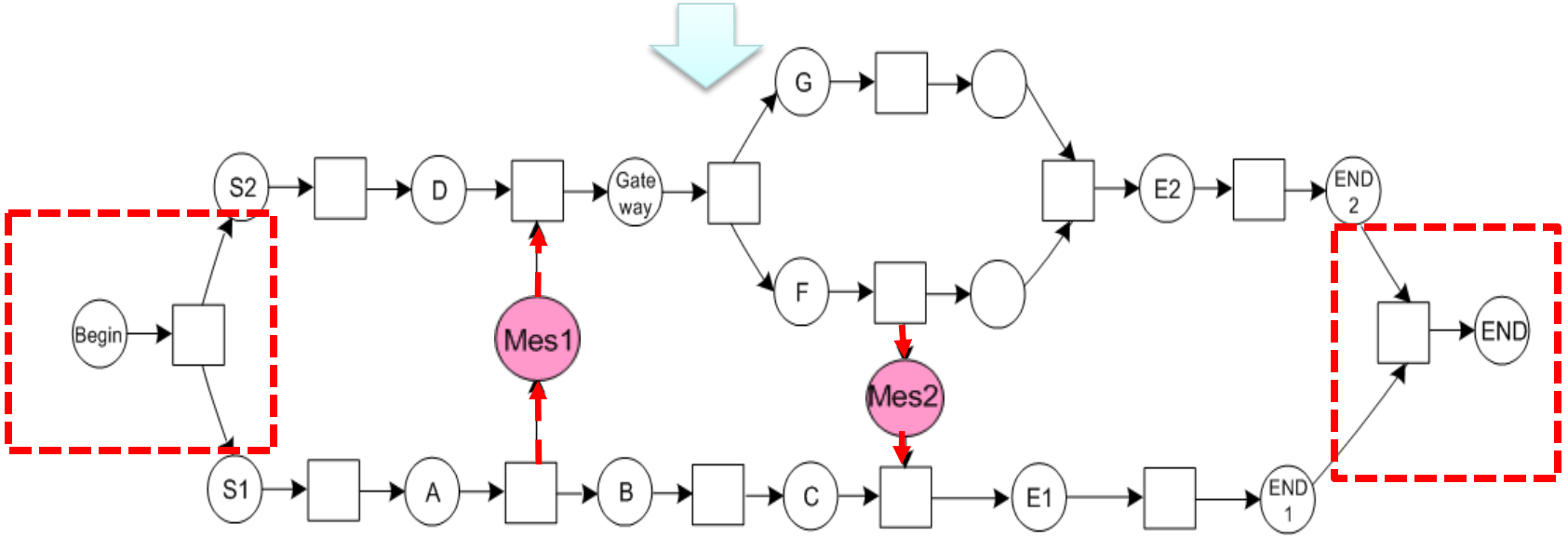
BPMN2Petri net映射

BPMN图元 －Flow Objects			Petri net
event	start		
	intermediate		
	end		
	task		
gateway	fork		
	join		
	decision		
	merge		

BPMN2Petri net映射

分析示例





模型安全性验证

- 将Petri网模型转换为有向图 G(V, E)
  - 顶点集V={M $(p_0, p_1, \cdots, p_n)$}是模型的状态集合
  - 有向弧E记录可执行变迁及其引起的状态迁移
- 验证属性
  - 可达性
    - 图搜索
  - 死锁
    - 查找可达图的叶子节点。若叶子节点的状态标识中存在token等于1的库所，且该库所不是结束库所
  - 环路
    - 验证方法：环路搜索算法

Q&A